

TRIBHUVAN UNIVERSITY
Institute of Science and Technology
2066

Bachelor Level/ First Year/ First Semester/ Science

Computer Science and Information Technology (MTH 104)

(Calculus and Analytical Geometry)

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

Full Marks: 80

Pass Marks: 32

Time: 3 hours.

Group A (10x2=20)

1. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.
2. Find the critical points of the function $f(x) = x^{3/2} (x - 4)$.
3. Does the following series converge?
$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$
4. Find the polar equation of the circle $(x + 2)^2 + y^2 = 4$.
5. Find the area of the parallelogram where vertices are A(0, 0), B(7, 3), C(9, 8) and D(2, 5).
6. Evaluate the integral
$$\int_t^{2t} \int_0^1 (\sin x + \cos y) dx dy.$$
7. Evaluate the limit
$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$$
8. Find $\left(\frac{\partial \omega}{\partial x}\right)_{y,z}$ if $\omega = x^2 + y - z + \sin t$ and $x + y = t$.
9. Solve the partial differential equation $p + q = x$.
10. Find the general integral of the linear partial differential equation $z(xp - yq) = z^2 - x^2$.

Group B (5x4=20)

11. State and prove Rolle's theorem.
12. Find the length of the cardioid $r = 1 + \cos \theta$.

13. Define unit tangent vector of a differentiable curve. Find the unit tangent vector of the curve $r(t) = (\cos t + t \sin t)i + (\sin t - t \cos t)j$, $t > 0$.

14. What do you mean by critical point of a function $f(x, y)$ in a region? Find local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

15. Find a particular integral of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 2y - x^2.$$

Group C (5x8=40)

16. Graph the function $y = x^{4/3} - 4x^{1/3}$.

17. What do you mean by Taylor's polynomial of order n ? Obtain Taylor's polynomial and Taylor's series generated by the function $f(x) = \cos x$ at $x = 0$.

18. Find the volume of the region enclosed by the surface $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

19. Obtain the absolute maximum and minimum values of the function

$f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by lines $x = 0, y = 0, y = 9 - x$.

OR

Evaluate the integral $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz dy dx$.

20. Show that the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, $c^2 = \frac{T}{\rho}$, is

$$u(x, t) = \frac{1}{2} [f(x + ct) + f(x - ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) ds$$

and deduce the result if the velocity is zero.

OR

Find a particular integral of the equation $(D^2 - D^1)z = A \cos(lx + my)$ where A, l, m are constants.